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Reply to Comment on the paper "Energy Loss of Charm Quarks in the Quark-Gluon Plasma: Collisional vs Radiative" by Mishra et al

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The comments raised in Ref. [1] by Mishra et al aim at two papers contained in Ref. [2]. We show that those comments on Ref. [2] pointed out by Mishra et al in Ref. [1] are not relevant and the concept used in Ref. [2] is consistent and in compliance with the classical approximation of the transport coefficients [3]. We would also like to note that most of the comments in Ref. [1] were meant for light quarks, but are not even appropriate for heavy quarks.

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The comments of Mishra et al in Ref. [1] are organised as a list of observations ("lacunas"). Here we reply to these observations one by one.

Lacuna 1: Notations

The notations and/or expressions can easily be understood depending upon the physical considerations (massless or massive quarks) they are applied to. In the case of high momenta (at least $p > 5$ GeV/c), there is, in fact, no significant difference between E and p, for a charm quark jet.

Lacuna 2: Fokker-Planck (FP) Equation

We begin with the one-dimensional FP equation [2, 3] as

$$\frac{\partial}{\partial t}D(p,t) = \frac{\partial}{\partial p}[\Gamma_1(p)D(p,t)] \ + \ \frac{\partial^2}{\partial^2 p}[\Gamma_2(p)D(p,t)], \tag{1}$$

that describes the evolution of the momentum distribution, $D(p,t)$, in a domain $(-\infty \leq p \leq \infty)$, at a given time t $(0 \leq t \leq \infty)$, of a test particle undergoing Brownian motion. $\Gamma_1(p)$ and $\Gamma_2(p)$ are known as the moments or the FP coefficients or the transport coefficients. Usually $\Gamma_1(p)$ is related to the collisions whereas $\Gamma_2(p)$ to the momentum diffusion in the medium when a test particle undergoes Brownian motion.

We now outline the classical approximations [2, 3] for the transport coefficients in which the drag force, $\mathcal{A}$, is assumed to be related to the collisional energy loss, $-dE/dL$, as

$$\Gamma_1(p) = -\frac{dE}{dL} \approx p\mathcal{A}(p) \approx p\mathcal{A}, \tag{2}$$

where $\mathcal{A} = \langle \mathcal{A}(p) \rangle = \langle -\frac{1}{p}\frac{dE}{dL} \rangle$, and the diffusion coefficient is related to the drag as

$$\Gamma_2(p) \approx p\mathcal{A}(p)p \approx T\mathcal{A}(p)p \approx T\langle -\frac{dE}{dL} \rangle \equiv \mathcal{D}_F. \tag{3}$$

Within this approximation the drag, $\mathcal{A}$, and the diffusion, $\mathcal{D}_F$, coefficients are momentum independent. Now one can write the FP equation in (1) as [3]

$$\frac{\partial}{\partial t}D(p,t) = \mathcal{A}\frac{\partial}{\partial p}[pD(p,t)] \ + \ \mathcal{D}_F\frac{\partial^2}{\partial^2 p}D(p,t). \tag{4}$$

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The authors of Ref. [1] also agree to the above form (see, e.g, eq. (C3) of Ref. [1]), which implies that they, obviously, accept the momentum independence approximation of $\mathcal{A}$ and $\mathcal{D}_F$.

We now quote the solution of the FP equation (4) for a given time t as obtained in Ref. [2]

$$D(p,t) \; = \; \frac{1}{\sqrt{\pi\,\mathcal{W}(t)}} \, \exp\left[-\frac{\left(p - p_0\, e^{-\int_0^t \mathcal{A}(t')\,dt'}\right)^2}{\mathcal{W}(t)} \right] \, , \tag{5}$$

where $\mathcal{W}(t)$ is given by

$$\mathcal{W}(t) = \left(4 \int_0^t \mathcal{D}_F(t') \exp\left[2 \int^{t'} \mathcal{A}(t'')\,dt'' \right] dt' \right) \left[\exp\left(-2 \int_0^t \mathcal{A}(t')\,dt' \right) \right] \, . \tag{6}$$

We further clarify that for a given initial condition, $\delta(p - p_0)$, and for the momentum independence approximation[1] of $\mathcal{A}$ and $\mathcal{D}_F$ the above solution is correct and unique which could easily be checked as the detailed calculational steps are given in Ref. [2]. It is also worthwhile to note that for massless quarks the analytically obtained solution of the FP equation in (4) agrees well with the solutions [5] if one solves (1) numerically for a given temperature ($T = 400$ MeV) and initial momentum ($p_0 = 16$ GeV/c), up to $t \sim 8$ fm/c, beyond which the classical approximations for transport coefficients are not valid. This time regime is quite appropriate to study the energy loss probability distribution of a jet in quark-gluon plasma (QGP) whose life time is of the same order for RHIC energies.

Now, the authors of Ref. [1] claim that there should be also a linear term in momentum, $(p - \langle p \rangle)$, in the exponential of (5) for reducing to the massless Maxwellian form in the thermodynamic limit. Within the given initial condition, $\delta(p - p_0)$, and the momentum independence approximation of $\mathcal{A}$ and $\mathcal{D}_F$, such a linear term in the exponent of (5) does not appear (see also Lacuna 4). Of course, if one changes the initial condition or relaxes the momentum independence of $\mathcal{A}$ and $\mathcal{D}_F$, one will then arrive at an altogether different solution. Note that the authors of Ref. [1] have not defined clearly what they really mean by "the possibility of a different type of solution". However, we intend to discuss various possibilities later in lacuna 4 while analysing the asymptotic forms of the FP equation in detail.

Lacuna 3: Probability function in p vs. E

According to (5) the function $D(p,t)$ is a probability distribution in p in one dimension ($-\infty \le p \le \infty$) for a given time t. Since it is an even function in p the normalisation condition can be written as

$$\int_{-\infty}^{\infty} D(p,t)dp = 2\int_0^{\infty} D(p,t)dp = 2\int_0^{\infty} f(E,t)dE = 1, \tag{7}$$

with $E = p = |p|$. Thus, the normalisation requirement, for the massless case, is unambiguously preserved. This also holds for charm quarks as long as we consider momenta p much larger than the charm quark mass.

Lacuna 4: Unphysical asymptotic form

Let us make another 'thought situation' where the drag ($\mathcal{A}$) and the diffusion ($\mathcal{D}_F$), are time independent, i.e., the test particle will experience a constant amount of drag and diffusion over a given length of medium or time traversed. This consideration is, however, without any loss of generality for the case of a static QGP.

In such a static medium the distribution in (5) reduces to

$$D(p,t) = \sqrt{\frac{\mathcal{A}}{2\pi\mathcal{D}_F\,(1 - e^{-2\mathcal{A}t})}} \; e^{-\frac{\mathcal{A}(p - p_0 e^{-\mathcal{A}t})^2}{2\mathcal{D}_F(1 - e^{-2\mathcal{A}t})}} \; . \tag{8}$$

Now in the thermodynamic limit ($t \to \infty$), the above equation simply becomes

$$D(p) = \sqrt{\frac{\mathcal{A}}{2\pi\mathcal{D}_F}} \; e^{-\frac{\mathcal{A}p^2}{2\mathcal{D}_F}} \; . \tag{9}$$

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[1] It is interesting to note that two of the present authors in Ref. [1] also used the same approximation and the said solution for a chemically

At this point, we must check the uniqueness of the solution in (5). For the purpose, we refer to the second pair of the characteristic equation in Ref. [2](See. as for example equation (22) in the first article in Ref. [2]) corresponding to (4) in Fourier space, which amounts to the $t \to \infty$ limit, that reads

$$\frac{\partial x}{\mathcal{A}x} = -\frac{\partial \tilde{D}}{\mathcal{D}_F x^2 \tilde{D}}, \tag{10}$$

where $\tilde{D} = \tilde{D}(x)$ is the Fourier transform of the momentum distribution function. It is easy to show that the solution obtained using (10) agrees perfectly with that of given in (9). So, the solution obtained in Ref. [2] is unique and a linear momentum term in the exponential does not arise, once the FP equation is given by (C3) of Ref. [1] or by (4). However, we are aware of the fact that relaxing our momentum independence approximation of $\mathcal{A}$ and $\mathcal{D}_F$, will certainly lead to different solutions, as the form of the FP equation will then be different from (4) or (C3) in Ref. [1]. We analyse various possibilities in the Appendix.

Based on our analysis in the Appendix we note that the FP coefficients, Γ_1 and Γ_2, cannot simply be assumed in any form to ensure the correct form of the (equilibrium) distribution because this requires at least an exact evaluation of the drag ($\mathcal{A}$) and the diffusion ($\mathcal{D}_F$) coefficients, which is even indeed a very involved task[2] [6]. But knowing the collisional energy loss in terms of elementary collision reaction amplitudes, one can approximate the drag as in (2) and the diffusion as in (3). The form of the approximation determines, a priori, the shape of the equilibrium distribution as discussed in the Appendix. Obviously, the momentum independence of $\mathcal{A}$ and $\mathcal{D}_F$ produces at least a consistent solution over the momentum range, $-\infty \le p \le \infty$, in contrast to the other possibilities (see Appendix). Again, we would also like to note that for small t the change in diffusion[3] will affect only the width and the height of the distribution, but not the peak position which is determined by the drag. Since we really do not wish to obtain an equilibrium distribution for a jet but intend to obtain an energy-loss probability distribution over a finite length ($\sim$ 8 to 10 fm) of the medium, (5) is a reasonably good approximation [3]. In addition, it was independently verified [5] numerically without any approximation as given in (1), where Γ_1 and Γ_2 are obtained using kinetic theory calculations. Nevertheless, it would have been easier for us to discuss this problem if the authors of Ref. [1] would have been more specific in their statement about "a different type of solution".

Now, the purpose of the study in Ref. [2] was the following: before we took up these studies [2], the dominance of the radiative energy loss for the phenomenon of jet quenching in heavy-ion collision seemed to be well established in the heavy-ion community. Within this simple approach we reconsidered, in 2003, the role of collisional partonic quenching and showed that it could be significant and cannot just be overlooked as it were done in the literature. However, only recently, after publication of new data on the nuclear suppression factor in RHIC, this simple idea has gained wider interest. In fact, an additional contribution to the partonic energy loss appears to be necessary and a collisional component is a welcomed remedy, as advocated in [7]. At this point, since RHIC BNL has provided very accurate data and LHC CERN will be operational soon, one indeed needs to improve our simple approach in different possible ways, which would definitely be a very desirable to verify the importance of the collisional energy loss.

Lacuna 5: Mean energy and loss

According to our explanations on the above three points (Lacuna 2-4) the calculation of the mean energy obtained in Ref. [2] follows from the expression of the distribution function itself in the classical approximation. The mean momentum of the test particle, as for example we refer to (8), is $\langle p \rangle = p_0 e^{-\mathcal{A}t}$, along with the diffusion process in momentum space as $\langle p^2 \rangle - \langle p \rangle^2 = 2\frac{\mathcal{D}_F}{\mathcal{A}}(1 - e^{-2\mathcal{A}t})$.

However, just to be sure, one can numerically compare $\langle p \rangle = p_0 e^{-\mathcal{A}t}$ with the one given in (C11) of Ref. [1]. They agree up to three to five decimal places for time interval $(1 - 10)$ fm/c as obviously the second term in (C11) is negligible and our classical approximation is consistent. This suggests that using $\langle p \rangle = p_0 e^{-\mathcal{A}t}$, is not a matter of great concern keeping in mind again the life-time of quark-gluon plasma. However, we note that it was used only for light quarks but proper care has been taken for heavy quarks.

Lacuna 6: Convolution integral for hadron spectrum

We could not make out the point raised by the authors of Ref [1] on our numerical calculations. In particular when they talk about a fixed length, L, in the convolution. However, the length $L(\phi)$ as defined in Ref. [2] will be

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[2] Here, we do not even wish to speak of the higher order FP coefficients in the Taylor expansion.

[3] For example, see the second reference in Ref. [2].

determined by a jet created at the transverse position, r and the production angle, ϕ in central collision. With this L one should convolute the spectra along with the geometry given by a cylinder of radius R in Bjorken hydrodynamics, which, however, should be restricted up to the critical temperature, $T_c \sim 0.2$ GeV. We checked our numerical results and reproduced $Q(p_T)$ for light hadrons. The results obtained for the scaled energy loss, $\frac{\Delta E}{E}$, for heavy quarks have been independently verified in Ref.[7]. Hence, we believe that the approach as well as the analytical and numerical solutions in Ref.[2] are correct as long as they are used within the domain of $(8-10)$ fm/c.

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APPENDIX A: ASYMPTOTIC SOLUTION OF FOKKER-PLANCK EQUATION WITH VARIOUS POSSIBILITIES OF ITS COEFFICIENTS:

For convenience we write down the general form of the FP equation from (1) for steady state as

$$\frac{\partial}{\partial t} D(p,t) = \frac{\partial}{\partial p}[\Gamma_1(p) D(p,t)] \; + \; \frac{\partial^2}{\partial^2 p}[\Gamma_2(p) D(p,t)] = 0. \tag{A1}$$

where $D(p,t)$ is, in general, the probability distribution in momentum, p $(-\infty \leq p \leq \infty)$ at a given time t $(0 \leq t \leq \infty)$ of a test particle. Now we investigate the asymptotic solutions of the FP equation with various choices of its coefficients, $\Gamma_1(p)$ in (2) and $\Gamma_2(p)$ in (3).

Type-I:

We consider $\Gamma_1(p) \approx p\mathcal{A}$ and $\Gamma_2(p) \approx \mathcal{A}p^2$. Now it is easy to show that the steady state solution of the FP equation in (A1) reads as

$$D(p) = \frac{C_1}{p^3} \ , \tag{A2}$$

where C_1 is a constant. This solution is not Maxwellian and diverges in the limit, $p \rightarrow 0$. Thus, it is not a consistent solution over $-\infty \leq p \leq \infty$.

Type-II:

We now consider $\Gamma_1(p) \approx p\mathcal{A}$ and $\Gamma_2(p) \approx \mathcal{A}Tp$. Similarly, the form of the steady state solution of the FP equation in (A1) can be obtained as

$$D(p) = \frac{C_2}{p} \ e^{-\frac{p}{T}} \ . \tag{A3}$$

This appears like a pseudo-Maxwellian form but is again not a consistent solution over $-\infty \leq p \leq \infty$ as it diverges in the limit, $p \rightarrow 0$, like the Type-I above. However, it is very easy to anticipate the reason.

Type-III (our case):

We consider $\Gamma_1(p) \approx p\mathcal{A}$ and $\Gamma_2 = \mathcal{D}_F$. The steady state solution is

$$D(p) = C_3 \ e^{-\frac{Ap^2}{2D_F}} \ , \tag{A4}$$

Type-IV :

We now explore the simplest one where both Γ_1 and Γ_2 are momentum independent. Then the steady state solution reads

$$D(p) = C_4 \, e^{-\frac{\Gamma_1}{\Gamma_2} p} \ . \tag{A5}$$

Now one can claim it to be of the Maxwellian form just by choosing $\frac{\Gamma_1}{\Gamma_2} = \frac{1}{T}$.

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